

Warm-up

Use your Alg II knowledge and intuition to attempt:

1) Solve 4

2) Solve

3) Solve

4) Solve

Log Functions Day 4:

- (2.13) Exponential Equations
- (2.10/2.13) Inverse of Exponential Functions

Homework:
Exponential Equations

Quiz Next Class!
(On days 1 - 3)

General approach to exponential equations

1. Isolate the exponential part
2. Take the log of both sides
3. Solve for x (distribute, rearrange, factor)

Example A:

$$4^{2x-3} - 3 = 137$$

$$\log_4(4^{2x-3}) = \log_4(140)$$

$$2x - 3 = \log_4 140$$

$$2x = 3 + \log_4 140$$

$$x = \frac{3 + \log_4 140}{2}$$

Example B+C

Example B

$$4e^{7x} + 10 = 22$$

$$4e^{7x} = 12$$

$$e^{7x} = 3$$

$$\ln(e^{7x}) = \ln(3)$$

$$7x = \ln 3$$

$$x = \frac{\ln(3)}{7} = \frac{1}{7} \ln 3 = \ln(\sqrt[7]{3})$$

Example C

$$4(3^{2x-3}) - 3 = 17$$

$$4(3^{2x-3}) = 20$$

$$\log_3(3^{2x-3}) = \log_3(5)$$

$$2x-3 = \log_3 5$$

$$x = \frac{\log_3 5 + 3}{2}$$

$$\frac{1}{2} \log_3 5 + \frac{3}{2}$$

$$\frac{1}{2} (\log_3 5 + 3)$$

$$\frac{1}{2} (\log_3 5 + \log_3 27)$$

$$\frac{1}{2} \left(\log_3 135 \right)$$

$$\log_3 \sqrt{135}$$

Example D+E

Example D

$$\frac{119}{e^{6x} + 14} = 7$$

$$119 = 7(e^{6x} + 14)$$

$$17 = e^{6x} + 14$$

$$3 = e^{6x}$$

$$\ln 3 = 6x$$

$$\frac{\ln 3}{6} = x$$

Example E

$$(8^{x+1})(2^{x-1}) = 10$$

$$((2^3)^{x+1})(2^{x-1}) = 10$$

$$(2^{3x+3})(2^{x-1}) = 10$$

$$\log_2(2^{4x+2}) = \log_2(10)$$

$$4x + 2 = \log_2 10$$

$$4x = -2 + \log_2 10$$

$$x = \frac{-2 + \log_2 10}{4}$$

Example F+G

Example F

$$e^{4x-5} + 10 = 12$$

Example G: Find the x -value of the point at which the graph of $f(x) = 9^{x-3}$ intersects the graph of $g(x) = 3^{x+5}$

Example H

$$2^{2x} - 5\overset{A}{\boxed{(2)^x}} + 4 = 0$$

$$(2^x)^2 - 5(2^x) + 4 = 0$$

$$A^2 - 5A + 4 = 0$$

$$(A - 4)(A - 1) = 0$$

$$\begin{aligned} A &= 4 \\ 2^x &= 4 \\ x &= 2 \end{aligned}$$

$$\begin{aligned} A &= 1 \\ 2^x &= 1 \\ x &= 0 \end{aligned}$$

Example I

$$9^{2x} + 6\underbrace{(9)^x}_A - 27 = 0$$

Let $f(x) = 9^{2x} + 6(9)^x - 27$

Find all input values in the domain of f that yield an output of 0

$$(9^x)^2 + 6(9^x) - 27 = 0$$

$$A^2 + 6A - 27 = 0$$

$$(A + 9)(A - 3) = 0$$

$$A = -9$$

$$\log_9(9^x) = \log_9(-9)$$

impossible

$$A = 3$$

$$\log_9(9^x) = \log_9(3)$$

$$x = \frac{1}{2}$$

Example J - Actual Question From 2025 AP Exam FRQ 4

The function m is given by $m(x) = e^{2x} - e^x - 12$.

Find all inputs in the domain of m that yield an output value of 0.

$$0 = e^{2x} - e^x - 12$$

Let $A = e^x$

$$0 = (e^x)^2 - e^x - 12$$

$$0 = A^2 - A - 12$$

$$0 = (A - 4)(A + 3)$$

$$A = 4$$

$$e^x = 4$$

$$x = \ln 4$$

$$A = -3$$

$$e^x = -3$$

Example K - Inverse Functions

Let $f(x) = 3(2)^x + 5$. Find an expression for $f^{-1}(x)$.

$$X = 3(2)^y + 5$$

$$X - 5 = 3(2)^y$$

$$\frac{X-5}{3} = (2)^y$$

$$\log_2\left(\frac{X-5}{3}\right) = \log_2(2^y)$$

$$\log_2\left(\frac{X-5}{3}\right) = y$$

$$f^{-1}(x) = \log_2\left(\frac{X-5}{3}\right)$$

Example L+M - Inverse Functions

Let $g(x) = 3^{x+4} - 10$. Find $g^{-1}(x)$.

Let $h(x) = e^{2x-5}$. Find $h^{-1}(x)$.

Example N

Let $f(x) = -5^{3x+4} + 7$

Find an expression for $f^{-1}(x)$

Example O - Calculator/Desmos

Let $f(x) = \frac{20}{e^{kx}+2}$. If $f(-2) = 5$, find k .

Given this value of k , find $f(1)$.

Example P - Calculator/Desmos

Let $g(x) = 100 - 3^{kx+1}$. If $g(2) = -56$, find k .

Given this value of k , find $g(-3)$.

$$\text{Solve } \log_4(4^{3x-1}) = \log_4(16)$$

$$3x-1 = 2$$

$$3x = 3$$

$$x = 1$$

$$\text{Solve } 4^{3x-1} = 16$$

$$\log_4(4^{3x-1}) = \log_4(4^2)$$

$$3x-1 = 2$$

$$\text{Solve } 4^{3x-1} = 16$$

$$\frac{4^{3x}}{4^1} = 16$$

$$\frac{4^{3x}}{4} = 16$$

$$4^{3x} = 64$$

$$(4^3)^x = 64$$

$$64^x = 64$$

$$x = 1$$

Solve $4^{3x-1} = 32$

$$(2^2)^{3x-1} = 2^5$$

$$2^{6x-2} = 2^5$$

$$6x-2 = 5$$

$$6x = 7$$

$$x = \frac{7}{6}$$

Solve $4^{3x-1} = 32$

$$(3x-1) \cdot \log_2 4 = 5$$

$$(3x-1) \cdot 2 = 5$$

$$6x-2 = 5$$



$$\text{Solve } \log_3(3^{2x-1}) = \log_3(9^{2x+1})$$

$$2x-1 = (2x+1) \cdot \log_3 9$$

$$2x-1 = (2x+1) \cdot 2$$

$$2x-1 = 4x+2$$

$$-3 = 2x$$

$$-\frac{3}{2} = x$$

$$\text{Solve } 3^{2x-1} = 9^{2x+1}$$

$$3^{2x-1} = (3^2)^{2x+1}$$

$$3^{2x-1} = 3^{4x+2}$$

$$2x-1 = 4x+2$$

Solve $\frac{5^x}{25^{x+1}} = 125$

$$\frac{5^x}{(5^2)^{x+1}} = 5^3$$

$$\frac{5^x}{5^{2x+2}} = 5^3$$

$$5^{x-(2x+2)} = 5^3$$

$$x - 2x - 2 = 3$$

$$-x - 2 = 3$$

$$-x = 5$$

$$x = -5$$

Solve $\frac{5^x}{25^{x+1}} = 125$

$$\log_5\left(\frac{5^x}{25^{x+1}}\right) = \log_5 125$$

$$\log_5(5^x) - \log_5(25^{x+1}) = 3$$

$$x - (x+1) \log_5 25 = 3$$

$$x - (x+1) \cdot 2 = 3$$

$$x - 2x - 2 = 3$$

